Modeling of Cap and Trade Programs

by

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I. INTRODUCTION OF THE CAP AND TRADE MODEL

A "Cap and Trade" (C&T) system has many desirable features for implementing pollution emission reductions. The cap limits emissions. The trading ensures that the reduction will be achieved at the lowest possible cost (economic efficiency). The initial allocation of permits can be used to address issues of fairness (equity).

The model we use for the C&T analysis has been previously developed and successfully applied to simulate the workings of interregional (and international) C&T systems. It is based on established economic principles (equilibrium and optimization). The model can be solved either as a system of simultaneous equations or as a non-linear programming model. It has been applied to the analysis of C&T associated with the Kyoto Protocol, emissions trading within the European Union, the Regional Greenhouse Gas Initiative (RGGI), ten EPA regions covering all states of the U.S, Midwestern Governors Association (MGA) region, Minnesota internal state trading, Western Climate Initiative (WCI), and Pacific Rim states and countries (see Rose et al., 1998; Rose and Zhang, 2004; Rose et al., 2006; CCS, 2008; Rose and Wei, 2008).

This model is based on the ability of unrestricted permit trading to achieve a cost-effective allocation of resources in the presence of externalities (see, e.g., Tietenberg, 2007). For permit purchasing states (or sectors), compliance costs are equal to own abatement cost plus the cost of permits, whereas for selling states (or sectors), compliance costs are equal to own abatement cost minus the revenues from selling permits. The model can readily be adapted to include such alternative design features as: variations in sector and source coverage, implications of the cap on emission reduction requirements over time, offsets, variations on auctioning, upstream vs. downstream application, borrowing and banking, and any explicit constraints on the permit price or trading (see Stevens and Rose, 2002; CCS, 2008). With a few modifications, the same model can also be used to simulate a carbon tax.

The model yields the following general results:

- GHG emission reductions (abatement and sequestration) for each entity (sector and/or state) before and after permit trading
- Cost (or cost savings) of GHG emission reductions for each trading entity before and after trading
- Number of permits traded (bought and sold) by each entity
- Equilibrium permit price
- Cost savings for each entity of joining the C&T program
- Auction revenues if the allowances are auctioned among trading entities instead of grandfathered

The model uses the following inputs (all the input data are collected from the state's Climate Change Action Plans):

- Projections of baseline GHG emissions for each trading entity
- Caps on GHG emissions for each entity (translated from the state reduction goals in target years)
- Marginal cost curve of GHG emission reduction for each entity based on the cost of all relevant mitigation/sequestration options

II. DEVELOPMENT OF MARGINAL COST CURVES

Many states have developed State Climate Change Action Plans. The following data are collected for each applicable mitigation option (that has been quantitatively analyzed) in these states:

- The range of the mitigation option's application (maximum percentage of total emissions that can be reduced by the option)

- The cost per ton of CO_2 that can be reduced (this is specified in terms of a cost-effectiveness, including the possibility of cost savings per unit GHG removed)

For each state, the mitigation options are then ordered from lowest cost to highest cost. A step function is developed based on the mitigation potential and cost per ton of CO_2 reduction for each policy option. Such a step function is illustrated in Figure 1. Next, a smooth curve is developed to fit the step function, which would be used as the marginal cost curve of the state in C&T policy analysis.

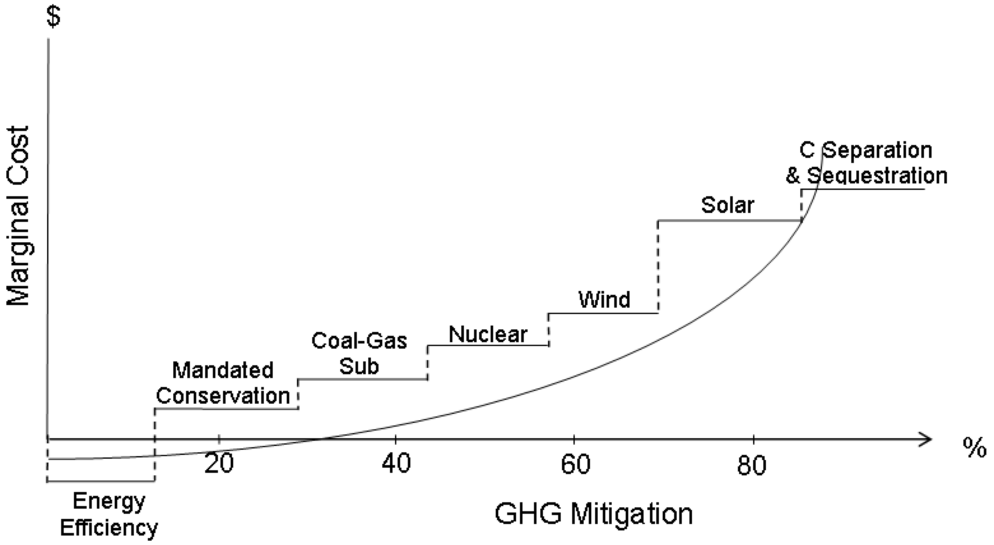

Figure 1. Illustrative Marginal Cost Step Function and Curve for GHG Mitigation

Prior CCS analysis for Minnesota can serve as an example of the construction of the mitigation marginal cost curve. Table 1 presents 8 example climate mitigation options out of the 37 options analyzed in a quantitative manner for Minnesota by CCS. Column 2 of the table presents the estimated 2025 annual GHG reduction potential for each option, with reduction potentials translated into percentages of the 2025 BAU emissions level in Column 4. The estimated cost or cost saving per ton of GHG removed by each option in 2025 is presented in Column 3. The options are ordered in ascending sequence in terms of cost, beginning with the cheapest option. Column 5 calculates the cumulative GHG reduction potentials of the first *n* policy options listed in the table. The last column presents the proportion of GHG mitigation contributed by each option.

Based on the data presented in Table 1, the stepwise marginal cost function for Minnesota in 2025 is first drawn in Figure 2. The horizontal axis represents the percentage of GHG emissions reduction, and the vertical axis represents the marginal cost or savings of mitigation. In the figure, each horizontal segment represents an individual mitigation option. The width of the segment indicates the GHG emission reduction potential of the option in percentage terms. The height of the segment relative to the x-axis shows the average cost (saving) of reducing one ton of GHG with the application of the option. The figure indicates that, collectively, the reduction potential of options from all economic sectors can avoid about 44% of 2025 baseline emissions in Minnesota. Our approach to develop the marginal cost curve based on state specific climate change action plans directly includes any introduction of new emission reduction technologies (such as carbon capture and storage) of the state. Furthermore, sensitivity analyses of mitigation options, for example, to account for different learning and penetration effects or technological innovations, can be readily reflected in the cost curve by variations in the width (usually lengthening) and height (usually lowering), as well as the sequencing of the corresponding segments of the options.

Next, we fit a smooth curve through the data using statistical analysis (see Figure 2). We weight each policy option based on its GHG mitigation potential to give relatively greater influence to those options that have the potential for higher levels of application. This fitted curve will then be used in our C&T analysis model.

Table 1. GHG Mitigation Options of Minnesota

Climate Mitigation Actions	Estimated 2025 Annual GHG Reduction Potential (MMtCO$_2$e)	Estimated Cost or Cost Savings per ton GHG Removed	GHG Reduction Potential as Percentage of 2025 Baseline Emissions[1]	Cumulative GHG Reduction Potential	Weights (add-up to 100)
……					
RCI-6: Non-Utility Strategies and Incentives To Encourage Energy Efficiency and Reduce GHG Emissions	1.3	-$37.00	0.65%	9.91%	1.48
……					
AFW-1: Agricultural Crop Management--A. Soil Carbon Management	1.3	-$2.00	0.65%	15.42%	1.48
TLU-5: Climate-Friendly Transportation Pricing / Pay as You Drive	2.1	-$1.00	1.05%	16.46%	2.39
AFW-8: End of Life Waste Management Practices--A. Landfilled Waste Methane	0.73	$1.00	0.36%	16.98%	0.83
AFW-4: Expanded Use of Biomass Feedstocks for Electricity, Heat, or Steam Production	3.8	$3.00	1.90%	18.87%	4.32
……					
ES-3: Efficiency Improvements, Repowering and other Upgrades to Existing Plants--Biomass co-firing	0.4	$12.00	0.20%	29.38%	0.46
AFW-5: Forestry Management Programs to Enhance GHG Benefits--A. Forestation	2.2	$13.00	1.11%	30.48%	2.50
……					
ES-5: Renewable and/or Environmental Portfolio Standard	15.7	$56.40	7.83%	43.53%	17.86
……					

[1] Minnesota 2025 projected consumption-based gross GHG emission level is 200.46 Million Metric Tons of CO$_2$e.

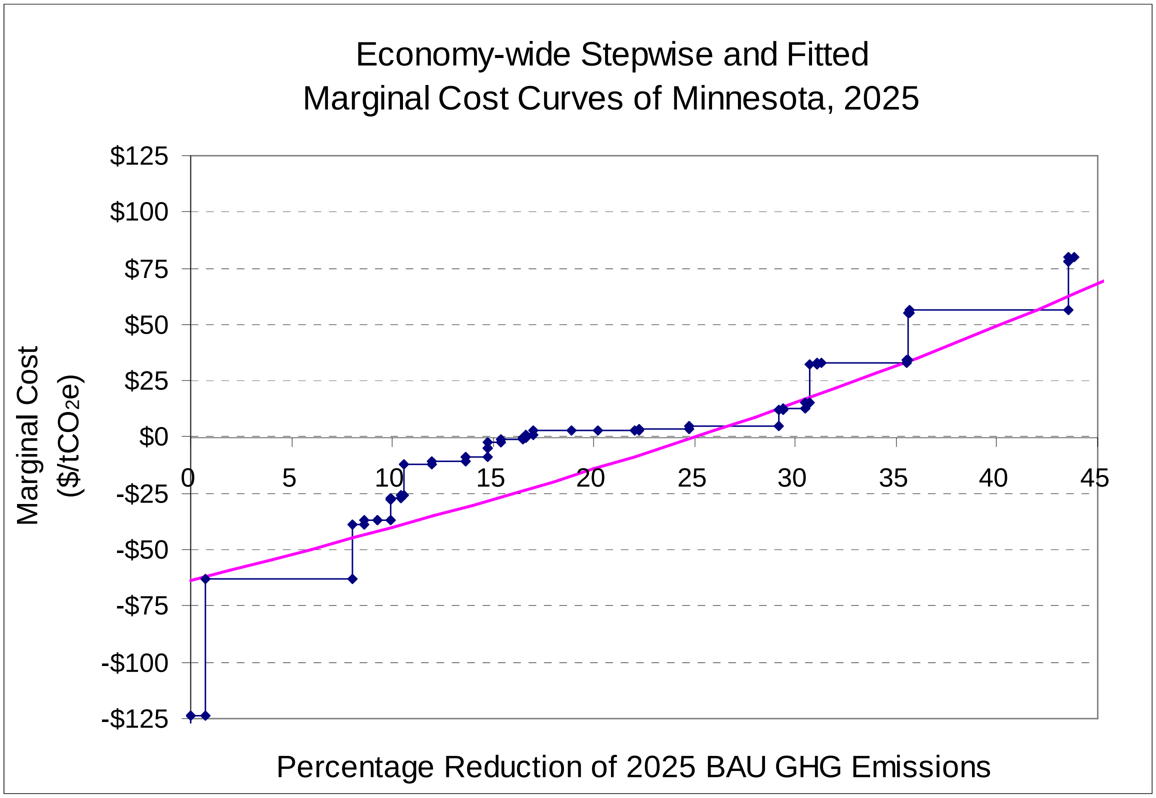

Figure 2. Stepwise and Fitted Marginal Cost Curve of Minnesota, 2025

The fitted curve shown in Figure 2 has the following functional form:

$$MC = a + b \times \ln(1 - R)$$

Where, *MC* is the marginal cost; *R* is the percentage reduction of GHG emissions; *a* and *b* are parameters.

The logarithmic functional form utilized here is consistent with theoretical expectations and empirical findings on diminishing returns of emission control (Nordhaus, 1991; 1994). As the emission reductions increase along the X axis, the cost to reduce one additional unit of emission is increasing in an accelerating speed.

The marginal cost curve for Minnesota has the following specification:

$$MC = -63.37 - 220.25 \times \ln(1 - R)$$

The fitted curve has an intercept with the Y-axis at MC = -$63.37. The curve increases to MC=0 at the emission reduction level of 25%, which indicates that Minnesota has cost-saving mitigation potentials (such as energy efficiency) up to the level of about 25% of the 2025 BAU emissions.

III. GENERAL ASSUMPTIONS ADOPTED IN THE ANALYSIS

The general assumptions we adopted in the C&T analysis and our modeling can be summarized as follows:

Emissions:

- All six GHGs — CO_2, methane (CH_4), nitrous oxide (N_2O), hydrofluorocarbons (HFCs), perfluorocarbons (PFCs), and sulfur hexafluoride (SF_6) — from the covered sectors are included in the analysis.
- The gross emissions (excluding forestry and agriculture soils sinks) are considered.

Marginal Cost Curves:

- Marginal cost curves embody direct mitigation costs only.
- Marginal cost curves do not include various transactions costs.
- Marginal cost curves do not distinguish between producer vs. consumer allocation of permits.
- For state that lacks direct cost data, the cost curve is approximated based on the data of one of its adjacent state that has quantified cost data available. We assume that the list of mitigation options for the adjacent state (state A) is applicable to the state without direct data (state B). Second, for state B, the estimated cost or cost savings per unit GHG removed for each option is assumed to be at the same level as of state A. Third, the mitigation potentials of each option are assumed to be proportional in each state; this requires that each option be adjusted by the ratio of emissions from the relevant sector of the two states. For example, if the emissions from the power sector are 50 $MMtCO_2e$ and 100 $MMtCO_2e$ in state A and state B, respectively, the mitigation potentials of the ES options for state A are multiplied by a factor of 2 (100/50=2) for application to state B.

Basic model (can be included in advanced versions):

- Offsets are not included.
- No safety valve (permit price limit) is included.
- Recycling of auction revenues (or tax revenues in the carbon tax cases) is not analyzed in the simulations.
- Banking and borrowing are not considered.

IV. SPECIFICATION OF THE CAP AND TRADE MODEL

The C&T model is based on well-established principles of the ability of unrestricted permit trading to achieve a cost-effective allocation of resources in the presence of externalities (see, e.g., Tietenberg, 2007). Where a strict cap implies unique GHG emission reduction requirements, the individual state and overall regional optimization can be accomplished without explicit consideration of the benefits side of the ledger (i.e., it yields “efficiency without optimality”). Therefore, the model simply requires equalization of marginal costs of all entities with the equilibrium permit price (see, Zhang, 2000; Loeschel and Zhang, 2002; Rose and Zhang, 2004). This ensures minimization of total net compliance costs for each state and minimization of total abatement costs for the region as a whole. For selling states (high cost states), they will reduce emissions up to the point where their marginal cost equals the prevailing market permit price, and accomplish their remaining reduction responsibility by purchasing available permits in the market. For purchasing states (low cost states), they would have the incentive to do more than their reduction targets indicate, so that they can sell their surplus permits on the open market to obtain profit. For the region as a whole, permit sales and purchases cancel out, simplifying the overall objective functions.

We assume that the marginal abatement cost function for state *i* is of the logarithmic form, similar to Nordhaus (1994):[1]

$$MC_i = a_i + b_i \times \ln(1 - R_i) \qquad i = 1\,,\ \ldots\ ,n \qquad (1)$$

where MC_i is the marginal cost of abatement for state *i*, R_i is the percentage of greenhouse gas abatement undertaken by state *i* in million tons of carbon dioxide equivalent (MMtCO2e), and a_i and b_i are cost parameters. This functional form has the desired property of positive and increasing marginal cost for $b_i < 0$. When $a_i = 0$, the cost curve starts from the origin. When $a_i < 0$, the curve can show the cost-saving mitigation range of the state. These cost parameters also capture technological and other distinctions that cause mitigation costs to differ across regions. By integration, the total cost of abatement for region *i*, TC_i, is:

$$TC_i = \int_0^{R_i} \left[a_i \cdot R_i - b_i \cdot (1 - R_i) \cdot \ln(1 - R_i) - b_i \cdot R_i \right] \cdot E_i \qquad i = 1\,, \ldots, n \qquad (2)$$

where E_i is each state's gross (unabated) emissions in MMtCO2e. Denoting the total required percentage reduction of emissions in region *i* in the absence of emissions trading as $\overline{R}_i$, the total abatement cost for each state in the absence of trading, $TC\overline{R}_i$, is calculated as:

$$TC\overline{R}_i = \int_0^{\overline{R}_i} \left[\left(a_i + b_i \cdot \ln(1 - r_i) \right) dr_i E_i \right] = \left[a_i \cdot \overline{R}_i - b_i \cdot (1 - \overline{R}_i) \cdot \ln(1 - \overline{R}_i) - b_i \cdot \overline{R}_i \right] \cdot E_i$$

$$i = 1\,, \ldots, n \qquad (3)$$

Emissions trading helps a region with relatively high marginal abatement cost to lower its compliance cost by avoiding the undertaking of autarkic actions. To minimize compliance costs, a purchasing state undertakes only some of its abatement requirement itself, $R_i E_i$, $\left(R_i E_i < \overline{R}_i E_i \right)$, up to the point where the marginal cost of doing so is equal to the endogenously determined permit price, *P*:

$$MC_i = a_i + b_i \times \ln(1 - R_i) = P \qquad i \in N \qquad (4)$$

where *N* is the set of all states.

The state meets the remaining demand, $\left(\overline{R}_i E_i - R_i E_i \right)$, via purchasing the "right to emit" at the regional market price, *P*. So, the total demand for emission permits of all purchasing states, *TD*, is:

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[1] The shape of the cost function for mitigating carbon emissions has been studied extensively. For example, Nordhaus (1994) found that the logarithmic functional form provided the best fit for the estimates of the marginal costs of mitigating a specific amount of carbon emissions among a number of economic modeling studies that he surveyed (a type of meta-analysis). Nordhaus (1994) used an analytical model to further derive a logarithmic relationship between the marginal costs and the percentage reduction.

$$TD = \sum_i \left(\overline{R}_i E_i - R_i E_i \right) \qquad i \in N \qquad (5)$$

On the other hand, for state *j*, with relatively low marginal cost, emissions trading provides it an incentive to undertake abatement and sell permits to those higher-cost states at the equilibrium permit price, *P*:

$$MC_j = a_j + b_j \times \ln(1 - R_j) = P \qquad j \in N \qquad (6)$$

The total amount of emissions permits available for sale in a given regional trading coalition *TS,* is:

$$TS = \sum_j \left(R_j E_j - \overline{R}_j E_j \right) \qquad j \in N \qquad (7)$$

The sum of total number of purchasing states *i* and total number of selling states *j* will be equal to *n*. At the equilibrium, the total demand for emissions permits in the region is equal to the total supply:

$$TD = TS \qquad (8)$$

Substituting Eq. (5) and Eq. (7) into Eq. (8) and rearranging terms yields the condition that the total emissions actually abated equal the total emission abatement requirement:

$$\sum_i R_i E_i = \sum_i \overline{R}_i E_i \qquad i = 1, \ldots, n \qquad (9)$$

We solve the model by minimizing total abatement costs of all states $\sum_i TC_i$ subject to Eq. (4), (6), and (9), using GAMS, an algebraic modeling system for linear, nonlinear, and integer programming problems (Brooke et al., 1996).[2] The solution yields the equilibrium permit price (*P*), each state's own abatement after trading $\left(R_i E_i\right)$, and each state's marginal abatement cost (MC_i). Because we focus on unrestricted emissions trading, in equilibrium the marginal cost of abatement for each region is the same and is equal to the permit price, indicated in Eq. (4) and Eq. (6).

This completes the description of the general model by which the permit price, MC_i, and $R_i E_i$ are determined endogenously in a competitive market. In the case where the permit price is set exogenously, as in the case of some auction-based C&T or the carbon tax cases, the situation becomes simpler because MC_i and hence $R_i E_i$ follows suit. There is no need for Eqs. (5), (7), (8), and (9) because the total sales of selling states to purchasing states are not equal to the total purchases, except by chance (when the specified permit price equals the equilibrium price).

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[2] The market equilibrium solution of our model is unique, so the same solution could be obtained without optimizing. The reason why we specify an objective function is that we use GAMS/MINOS, a solver mainly for optimization problems. The minimization of the total cost is a logical choice for an objective in the case of "cost-effectiveness" analysis here (i.e., when a policy target is set and decision units seek to attain it at least cost). Had we used a software package that is specifically designed to solve a simultaneous equation system, then there would have been no need for an objective function.

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